Opérateurs de réordonnement et polynômes de Grothendieck

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Séminaire du LIAFA - 12 janvier 2012

Opérateurs $\{\pi_i, 1 \leq i \leq n-1\}$

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vérifient les relations de tresse et des relations quadratiques :

$$\begin{aligned}
&\pi_i\pi_j = \pi_j\pi_i, && \text{si } |i-j| > 1,\\
&\pi_{i+1}\pi_i\pi_{i+1} = \pi_i\pi_{i+1}\pi_i, && \text{si } 1 \leq i \leq n-2,\\
&\pi_i\pi_i = \pi_i
\end{aligned}$$

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\hat{\pi}_{i+1}\hat{\pi}_i\hat{\pi}_{i+1} &= (\pi_{i+1} - 1)(\pi_i - 1)(\pi_{i+1} - 1) \\
&= \hat{\pi}_i\hat{\pi}_{i+1}\hat{\pi}_i, && \text{si } 1 \leq i \leq n-2,
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$$K_\omega = \hat{K}_\omega$$

$$\omega = [n, n-1, \ldots, 2, 1]$$

Générateurs de l'algèbre de 0-Hecke

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$\mathcal{H}(t_1, t_2)$ engendrée par les opérateurs $\{T_i, 1 \leq i \leq n-1\}$

$$
\begin{aligned}
&T_i T_j = T_j T_i, && \text{si } |i-j| > 1,\\
&T_{i+1} T_i T_{i+1} = T_i T_{i+1} T_i, && \text{si } 1 \leq i \leq n-2,\\
&(T_i - t_1)(T_i - t_2) = 0
\end{aligned}
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base : $(T_\sigma)_{\sigma \in \mathfrak{S}_n}$

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$$t_1 = -1, t_2 = 0 \longrightarrow \hat{\pi}_i$$

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2 bases : $(\pi_\sigma)_{\sigma \in \mathfrak{S}_n}$ et $(\hat{\pi}_\sigma)_{\sigma \in \mathfrak{S}_n}$

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$$\pi_\sigma = \pi_1 \pi_2 \pi_1$$

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&= \hat{\pi}_1\hat{\pi}_2\hat{\pi}_1 + \hat{\pi}_1\hat{\pi}_2 + \hat{\pi}_2\hat{\pi}_1 + \hat{\pi}_2 + \hat{\pi}_1 + 1
\end{aligned}
$$

Changement de base ?

$$\begin{aligned}
\sigma &= s_{i_1} \dots s_{i_m} \\
\pi_\sigma &= \pi_{i_1} \dots \pi_{i_m} \\
&= (\hat{\pi}_{i_1} + 1)(\hat{\pi}_{i_2} + 1) \dots (\hat{\pi}_{i_m} + 1) \\
&= \sum_{\mu} \hat{\pi}_\mu
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où une décomposition réduite de μ est un sous-mot d'une décomposition réduite de σ.(Lascoux)

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$\rightarrow \mu \leq \sigma$ pour l'ordre de Bruhat sur les permutations

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Un ordre sur les permutations, gradué selon le nombre d'inversions.

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Soit $\sigma \in \mathfrak{S}_n$, μ est un successeur de σ dans l'ordre de Bruhat ssi :

- $\mu = \sigma\tau$ avec τ transposition
- $\ell(\mu) = \ell(\sigma) + 1$

Exemple :

$$352614$$

$$\underline{53}2614 \quad \underline{4}5261\underline{3} \quad 3\underline{6}2\underline{5}14 \quad 35\underline{62}14 \quad 35\underline{4}61\underline{2} \quad 3526\underline{41}$$

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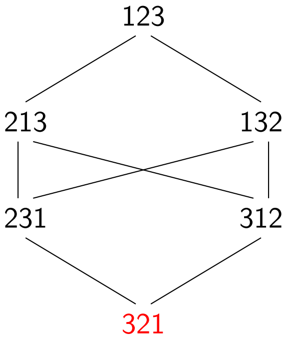
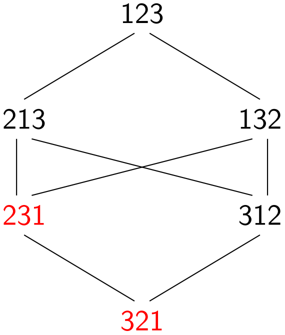
Successeurs : $..b...d.. \rightarrow ..d...b..$ avec entre b et d, uniquement des valeurs $< b$ ou $> d$

Changement de base : K, $\hat{K}$



$$\hat{K}_{321} = K_{\color{red}{321}}$$

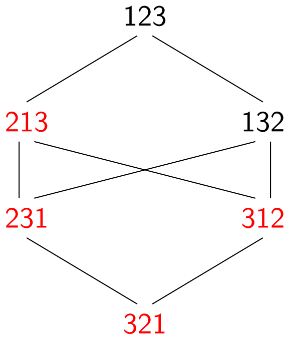
Changement de base : K, $\hat{K}$



$$\hat{K}_{321} = K_{321}$$

$$\begin{aligned}\hat{K}_{231} &= \hat{K}_{321}\hat{\pi}_1 = K_{321}(\pi_1 - 1)\\ &= K_{231} - K_{321}\end{aligned}$$

Changement de base : K, $\hat{K}$



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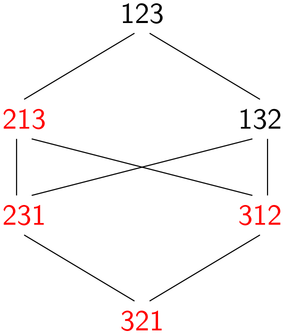
&= K_{231} - K_{321} \\
\hat{K}_{213} &= \hat{K}_{231}\hat{\pi}_2 = K_{231}\hat{\pi}_2 - K_{321}\hat{\pi}_2 \\
&= K_{213} - K_{231} - K_{312} + K_{321}
\end{aligned}
$$

Changement de base : K, $\hat{K}$



$$\hat{K}_{321} = K_{321}$$

$$\hat{K}_{231} = \hat{K}_{321}\hat{\pi}_1 = K_{321}(\pi_1 - 1) = K_{231} - K_{321}$$

$$\hat{K}_{213} = \hat{K}_{231}\hat{\pi}_2 = K_{231}\hat{\pi}_2 - K_{321}\hat{\pi}_2 = K_{213} - K_{231} - K_{312} + K_{321}$$

$$\hat{K}_\sigma = \sum_{\mu \geq \sigma} (-1)^{\ell(\mu) - \ell(\sigma)} K_\mu$$

$$K_\sigma = \sum_{\mu \geq \sigma} \hat{K}_\mu$$

Produits de π et $\hat{\pi}$

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Étude d'un produit particulier

Motivations géométriques : polynômes de Grothendieck, caractères de Demazure

Polynômes clés et polynômes de Grothendieck

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Polynômes dominants

$$G_{(\omega)} := \prod_{\substack{i=1\ldots n \\ j=1\ldots n-i}} (1 - y_j x_i^{-1}), \qquad \omega = [n, n-1, \ldots, 1]$$

Polynômes clés et polynômes de Grothendieck

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$$K_\lambda = \hat{K}_\lambda := x_1^{\lambda_1} x_2^{\lambda_2} \ldots x_n^{\lambda_n}, \qquad \lambda = \lambda_1 \geq \lambda_2 \geq \ldots \lambda_n$$

Action de π et $\hat{\pi}$

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$$f\pi_i = \frac{x_i.f - x_{i+1}.f^{s_i}}{x_i - x_{i+1}}$$

$$f\hat{\pi}_i = f(\pi_i - 1) = \frac{x_{i+1}.f - x_{i+1}.f^{s_i}}{x_i - x_{i+1}}$$

Bases de l'anneau des polynômes

On obtient l'ensemble des polynômes à partir des polynômes dominants.

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$$G_{(\sigma s_i)} := G_{(\sigma)}\pi_i,$$

si $\sigma(i) > \sigma(i+1)$.

$$\begin{tikzcd}
& G_{(123)} & \\
G_{(213)} \arrow[ur, "\pi_1"] & & G_{(132)} \arrow[ul, "\pi_2"'] \\
G_{(231)} \arrow[u, "\pi_2"] & & G_{(312)} \arrow[u, "\pi_1"'] \\
& G_{(321)} \arrow[ul, "\pi_1"] \arrow[ur, "\pi_2"'] &
\end{tikzcd}$$

Produit de polynômes de Grothendieck

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Theorème

(Lascoux)

$$G_{(\sigma)} \frac{y_{\sigma_1} \cdots y_{\sigma_k}}{x_1 \cdots x_k} \equiv G_{(\omega)} \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma}.$$

où $\sigma \in \mathfrak{S}_{\mathfrak{n}}$, $1 \leq k < n$, ζ un représentant particulier de la classe de σ dans $\mathfrak{S}_n / (\mathfrak{S}_k \times \mathfrak{S}_{n-k})$.

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$$\left.\begin{array}{l} \sigma = 43678215 \\ k = 3 \end{array}\right\} \zeta = 643|87521$$

Résolution du problème dans les bases K et $\hat{K}$.

$$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma}$$

$$K_\omega \hat{\pi}_{\omega\zeta} =$$

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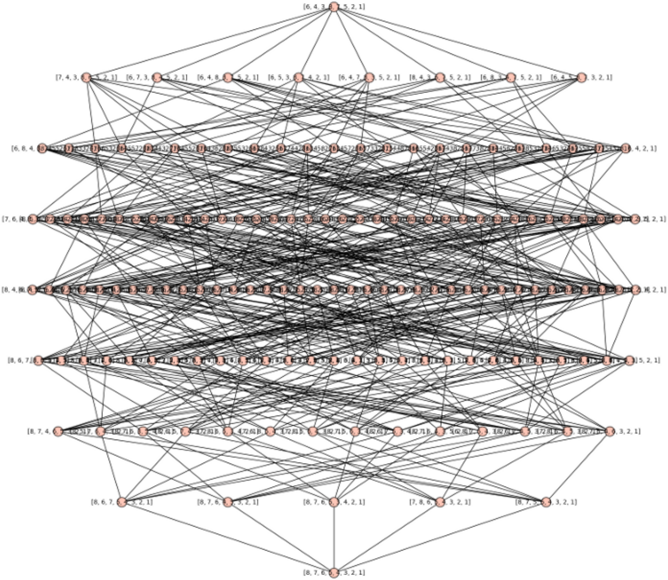
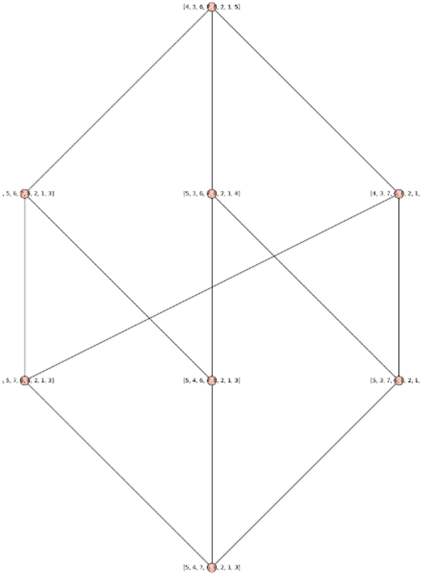
(152 termes)



$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} =$
(8 termes)



$\sigma = 43678215$

K_ω

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$\sigma = 43678215$

$K_\omega \hat{\pi}_{\omega\zeta}$

$\zeta = 643|87521$

s_3

64837521

87653421

s_5

$\omega = 87654321$

$\sigma = 43678215$

s_2

46378215

64387251

s_6

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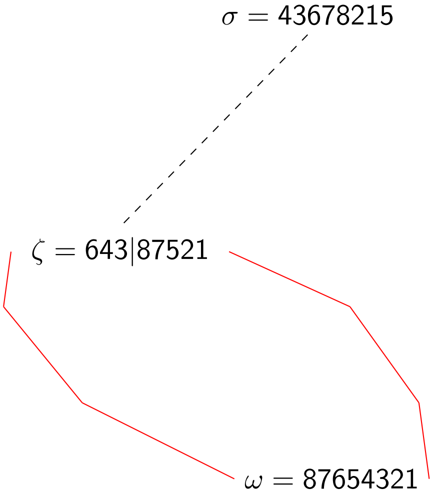

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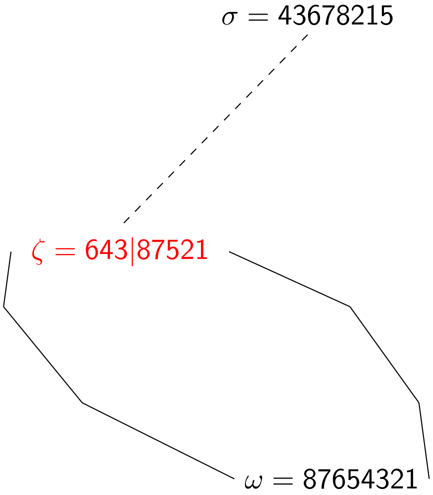
$$K_\omega \hat{\pi}_{\omega\zeta} = \sum_{\mu \geq \zeta} \pm K_\mu$$



$$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} = ?$$

$$\begin{aligned} K_\omega \hat{\pi}_{\omega\zeta} &= \sum_{\mu \geq \zeta} \pm K_\mu \\ &= \hat{K}_\zeta \end{aligned}$$

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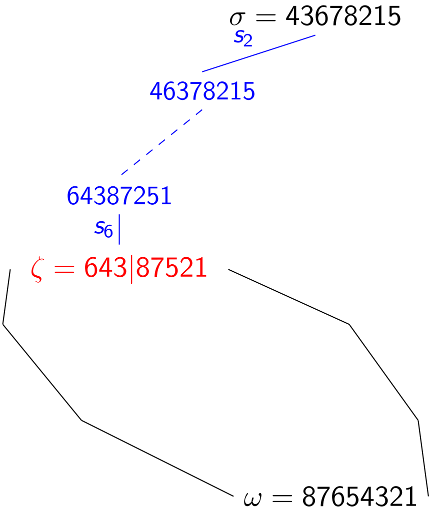
$\color{red}{\hat{K}_\zeta}$



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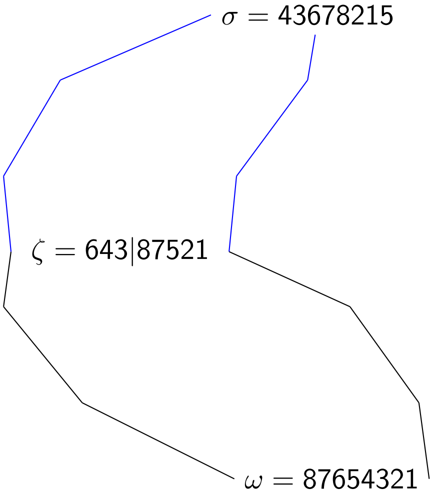
$$\hat{K}_\zeta \pi_{\zeta^{-1}\sigma}$$



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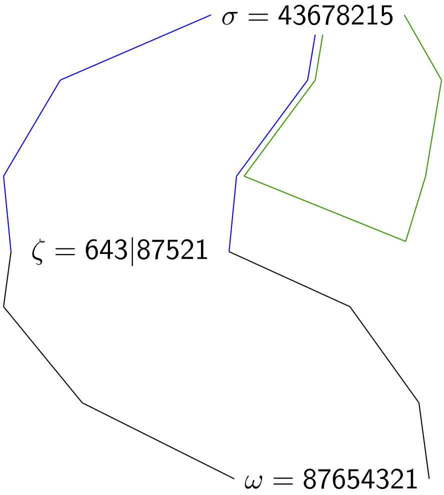
$$\begin{aligned} K_\omega \hat{\pi}_{\omega\zeta} &= \sum_{\mu \geq \zeta} \pm K_\mu \\ &= \hat{K}_\zeta \end{aligned}$$

$$\hat{K}_\zeta \pi_{\zeta^{-1}\sigma} = \sum_{\sigma \leq \mu \leq \zeta} \hat{K}_\mu$$



$$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} = ?$$

$$\begin{aligned} K_\omega \hat{\pi}_{\omega\zeta} &= \sum_{\mu \geq \zeta} \pm K_\mu \\ &= \hat{K}_\zeta \\ \hat{K}_\zeta \pi_{\zeta^{-1}\sigma} &= \sum_{\sigma \leq \mu \leq \zeta} \hat{K}_\mu \\ &= \sum_{\nu} \pm K_\nu \end{aligned}$$

$$436|78215$$

$\underline{6}3\underline{4}|78215$ $4\underline{63}|78215$ $436|\underline{87}215$ $436|78\underline{5}1\underline{2}$ $436|782\underline{51}$ $\underline{5}36|7821\underline{4}$ $4\underline{5}6|7821\underline{3}$ $43\underline{7}|\underline{6}8215$

$$
\begin{array}{c}
436|78215 \\
\underline{6}3\underline{4}|78215 \quad 4\underline{63}|78215 \quad 436|\underline{87}215 \quad 436|78\underline{5}1\underline{2} \quad 436|782\underline{51} \quad \underline{5}36|7821\underline{4} \quad 4\underline{5}6|7821\underline{3} \quad 43\underline{7}|\underline{6}8215
\end{array}
$$

$$\hat{K}_\zeta \pi_{\zeta^{-1}\sigma} = \sum_{\sigma \leq \mu \leq \zeta} \hat{K}_\mu$$

$$436|78215$$

$$\underline{6}3\underline{4}|78215 \quad 4\underline{63}|78215 \quad 436|\underline{87}215 \quad 436|78\underline{5}1\underline{2} \quad 436|782\underline{51} \quad \underline{5}36|7821\underline{4} \quad 4\underline{5}6|7821\underline{3} \quad 43\underline{7}|\underline{6}8215$$

$$\begin{aligned}\hat{K}_\zeta \pi_{\zeta^{-1}\sigma} &= \sum_{\sigma \leq \mu \leq \zeta} \hat{K}_\mu \\ &= \sum_{\substack{\nu \geq \sigma \\ \nu \ngeq \mu \\ \sigma < \mu \leq \zeta}} \pm K_\nu\end{aligned}$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

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$$\sigma = 1362|54$$

$$W_\sigma =$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1362|54$$

$$W_\sigma =$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1\textcolor{red}{3}62|54$$

$$W_\sigma =$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1362|54$$

$$W_\sigma = ((2, 6)$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1362|54$$

$$W_\sigma = ((2,6),(2,5)$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 136{\color{red}2}|54$$

$$W_\sigma = ((2,6),(2,5)$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1362|54$$

$$W_\sigma = ((2,6),(2,5),(4,6)$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1362|54$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = {\color{red}1}362|54$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5)$$

Soient σ, k, $W_\sigma(k)$ est la liste des k-transpositions de Bruhat (a, b) de σ par ordre décroissant sur $\sigma(a)$ puis croissant sur $\sigma(b)$.

$$\sigma = 1362|54$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$W_\sigma = (\tau_1, \tau_2, \ldots, \tau_m)$$

$$W_\sigma = (\tau_1, \tau_2, \ldots, \tau_m)$$

$$\mathfrak{E}_\sigma := K_\sigma \cdot (1 - \tau_1) \cdot (1 - \tau_2) \cdots (1 - \tau_m)$$

avec :

$$K_\mu \cdot \tau = \begin{cases} K_{\mu\tau} \text{ si } \tau \text{ est une transposition de Bruhat de } \mu, \\ 0 \text{ sinon.} \end{cases}$$

$$+K_{1362|54}$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$+K_{1362|54}$$

(2,6)

$$-K_{1462|53}$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$+K_{1362|54}$$

(2,6)

$$-K_{1{\color{red}4}62|{\color{red}5}3}$$

(2,5)

$$+K_{1{\color{red}5}62|{\color{red}4}3}$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$+K_{1362|54}$$

$$\downarrow (2,6)$$

$$-K_{1462|53}$$

$$\downarrow (2,5)$$

$$+K_{156\textcolor{red}{2}|4\textcolor{red}{3}}$$

$$\downarrow (4,6)$$

$$-K_{156\textcolor{red}{3}|4\textcolor{red}{2}}$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$+K_{1362|54}$

$(2,6)$

$-K_{1462|53}$

$(2,5)$

$+K_{1562|43}$

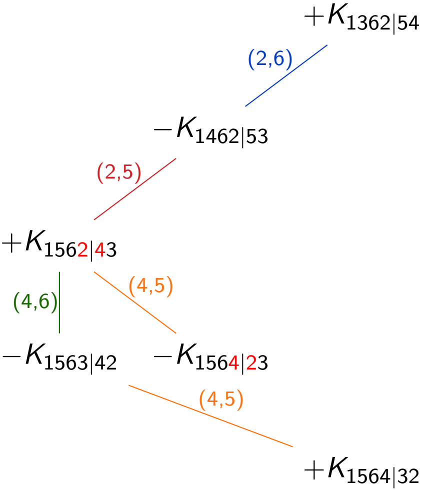
$(4,6)$

$-K_{1563|42}$

$(4,5)$

$+K_{1564|32}$

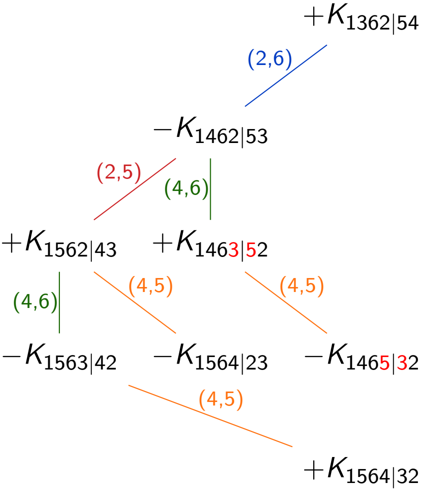
$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



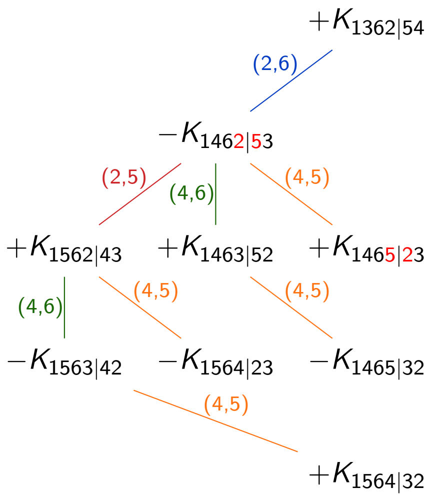
$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$
\begin{tikzcd}
 & & +K_{1362|54} \arrow[dl, dash, "(2,6)"'] \\
 & -K_{1462|53} \arrow[dl, dash, "(2,5)"'] \arrow[d, dash, "(4,6)"'] & \\
+K_{1562|43} \arrow[d, dash, "(4,6)"'] \arrow[dr, dash, "(4,5)"] & +K_{1463|52} & \\
-K_{1563|42} \arrow[drr, dash, "(4,5)"] & -K_{1564|23} & \\
 & & +K_{1564|32}
\end{tikzcd}
$$

$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$\begin{tikzcd}
 & & +K_{1362|54} \arrow[dl, dash, "{(2,6)}"'] \arrow[d, dash, "{(2,5)}"'] \\
 & -K_{1462|53} \arrow[dl, dash, "{(2,5)}"'] \arrow[d, dash, "{(4,6)}"'] \arrow[dr, dash, "{(4,5)}"] & -K_{1562|34} \\
+K_{1562|43} \arrow[d, dash, "{(4,6)}"'] \arrow[dr, dash, "{(4,5)}"] & +K_{1463|52} \arrow[dr, dash, "{(4,5)}"] & +K_{1465|23} \\
-K_{1563|42} \arrow[drr, dash, "{(4,5)}"] & -K_{1564|23} & -K_{1465|32} \\
 & & +K_{1564|32}
\end{tikzcd}$$

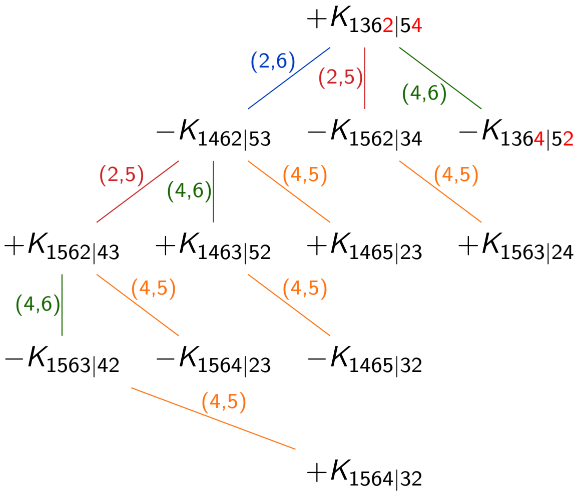
$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$
\begin{tikzcd}[arrows=dash]
 & & +K_{1362|54} \arrow[dl, "{(2,6)}"'] \arrow[d, "{(2,5)}"'] \\
 & -K_{1462|53} \arrow[dl, "{(2,5)}"'] \arrow[d, "{(4,6)}"'] \arrow[dr, "{(4,5)}"] & -K_{156{\color{red}2}|3{\color{red}4}} \\
+K_{1562|43} \arrow[d, "{(4,6)}"'] \arrow[dr, "{(4,5)}"] & +K_{1463|52} \arrow[dr, "{(4,5)}"] & +K_{1465|23} \\
-K_{1563|42} \arrow[drr, "{(4,5)}"] & -K_{1564|23} & -K_{1465|32} \\
 & & +K_{1564|32}
\end{tikzcd}
$$

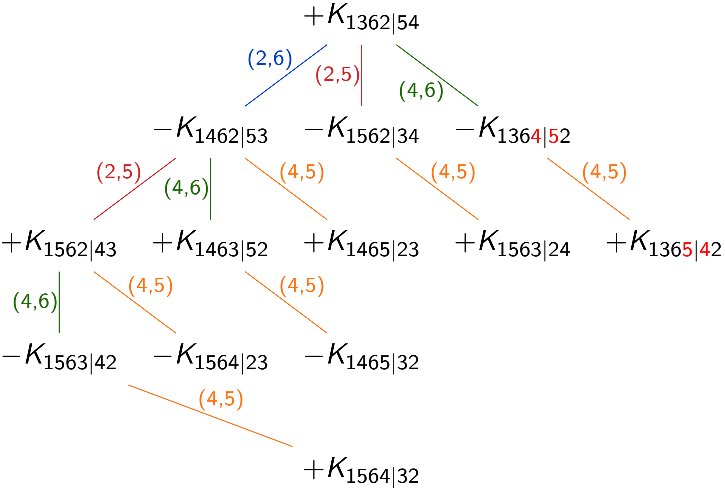
$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

$$
\begin{tikzcd}[column sep=small]
 & & +K_{1362|54} \arrow[dl, dash, "(2{,}6)"'] \arrow[d, dash, "(2{,}5)"] & \\
 & -K_{1462|53} \arrow[dl, dash, "(2{,}5)"'] \arrow[d, dash, "(4{,}6)"] \arrow[dr, dash, "(4{,}5)"] & -K_{1562|34} \arrow[dr, dash, "(4{,}5)"] & \\
+K_{1562|43} \arrow[d, dash, "(4{,}6)"'] \arrow[dr, dash, "(4{,}5)"] & +K_{1463|52} \arrow[dr, dash, "(4{,}5)"] & +K_{1465|23} & +K_{1563|24} \\
-K_{1563|42} \arrow[drr, dash, "(4{,}5)"] & -K_{1564|23} & -K_{1465|32} & \\
 & & +K_{1564|32} &
\end{tikzcd}
$$

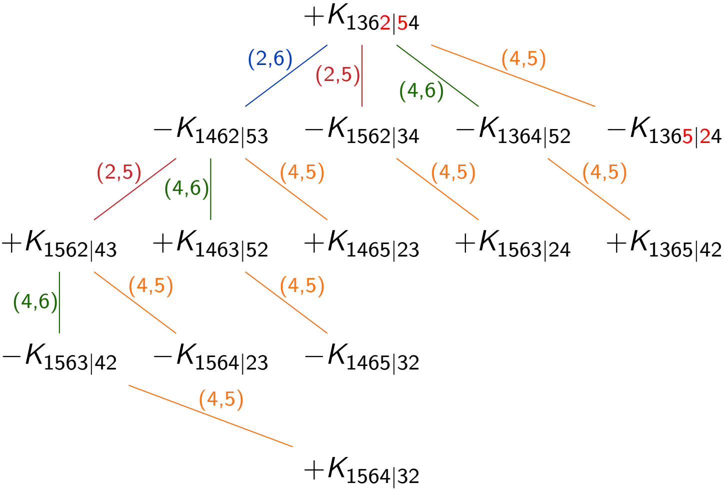
$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



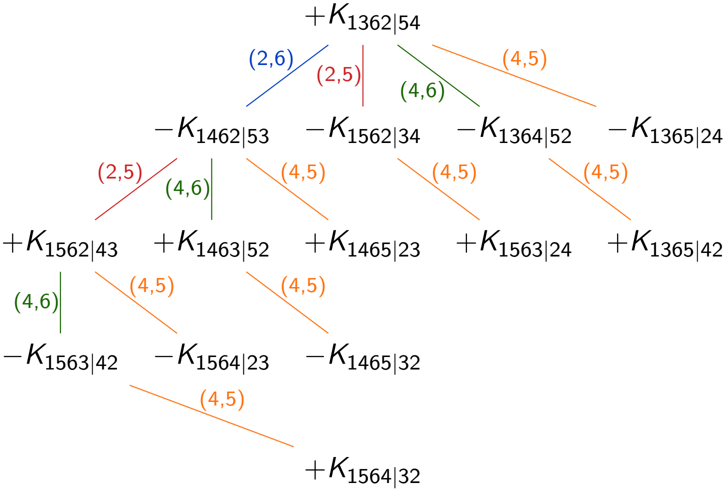
$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$



$$W_\sigma = ((2,6),(2,5),(4,6),(4,5))$$

Résultat :

$$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} = \mathfrak{E}_\sigma$$

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Démonstration :

$$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} = \left(\sum_{\mu \geq \zeta} \pm K_\mu \right) \pi_{\zeta^{-1}\sigma}$$

Résultat :

$$K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} = \mathfrak{E}_\sigma$$

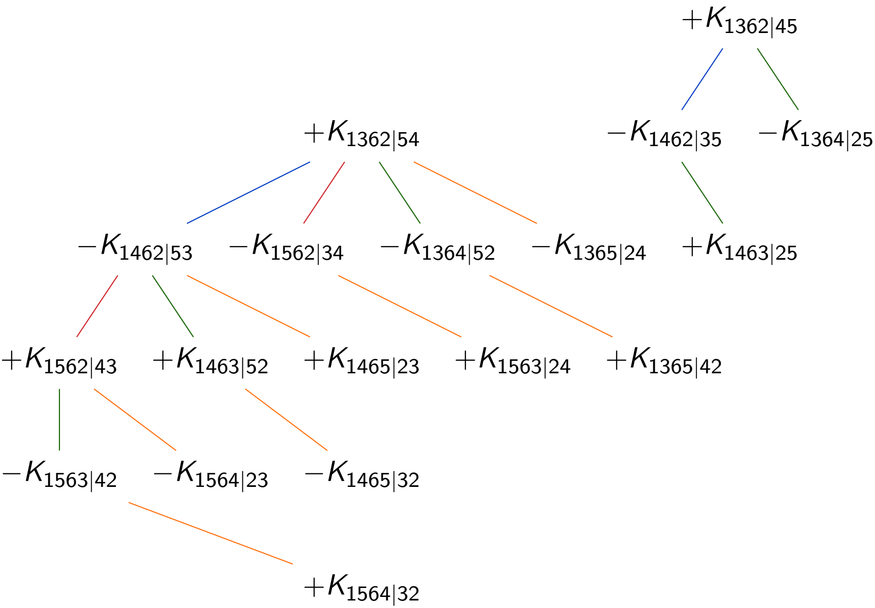
Démonstration :

$$\begin{aligned} K_\omega \hat{\pi}_{\omega\zeta} \pi_{\zeta^{-1}\sigma} &= \left(\sum_{\mu \geq \zeta} \pm K_\mu \right) \pi_{\zeta^{-1}\sigma} \\ &= \mathfrak{E}_\zeta \pi_{\zeta^{-1}\sigma} \end{aligned}$$

Lemme

Si $\sigma(i) > \sigma(i+1)$ *et* $i \neq k$, *alors*

$$\mathfrak{E}_\sigma \pi_i = \mathfrak{E}_{\sigma s_i}$$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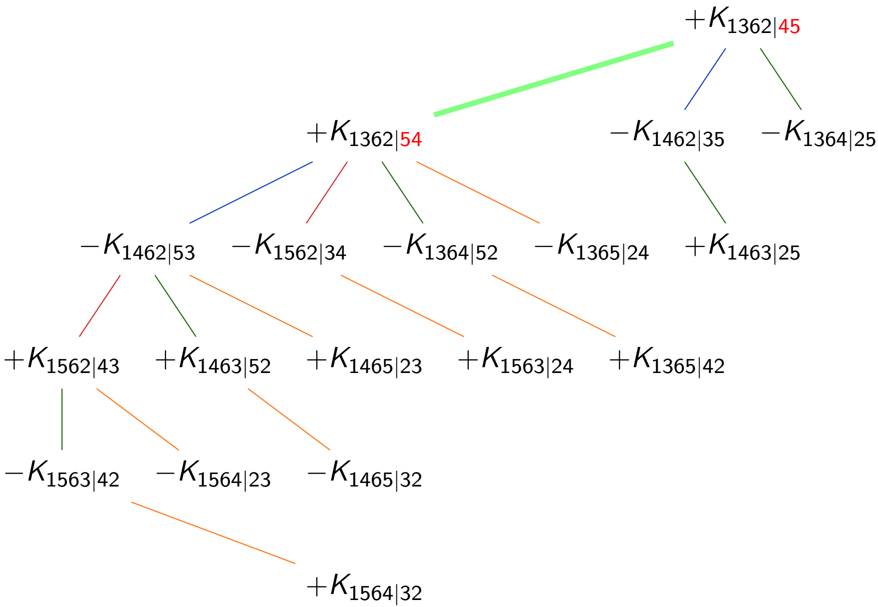
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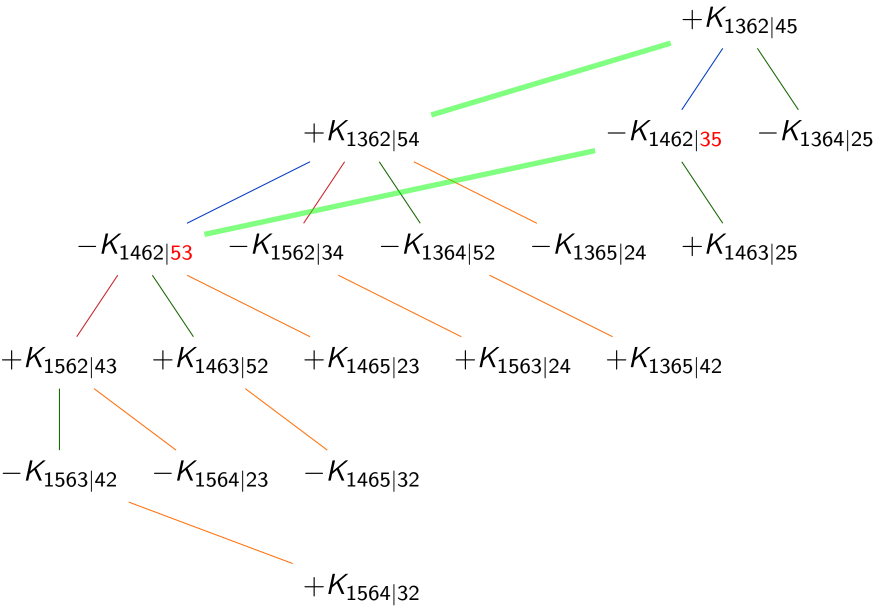
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$+K_{1362|45}$

$+K_{1362|54}$ $\quad -K_{1462|35}$ $\quad -K_{1364|25}$

$-K_{1462|53}$ $\quad -K_{1562|34}$ $\quad -K_{1364|52}$ $\quad -K_{1365|24}$ $\quad +K_{1463|25}$

$+K_{1562|43}$ $\quad +K_{1463|52}$ $\quad +K_{1465|23}$ $\quad +K_{1563|24}$ $\quad +K_{1365|42}$

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$+K_{1564|32}$

$+K_{1362|45}$

$+K_{1362|54}$ $\quad -K_{1462|35}$ $\quad -K_{1364|25}$

$-K_{1462|53}$ $\quad -K_{1562|34}$ $\quad -K_{1364|52}$ $\quad -K_{1365|24}$ $\quad +K_{1463|25}$

$+K_{1562|43}$ $\quad +K_{1463|52}$ $\quad +K_{1465|23}$ $\quad +K_{1563|24}$ $\quad +K_{1365|42}$

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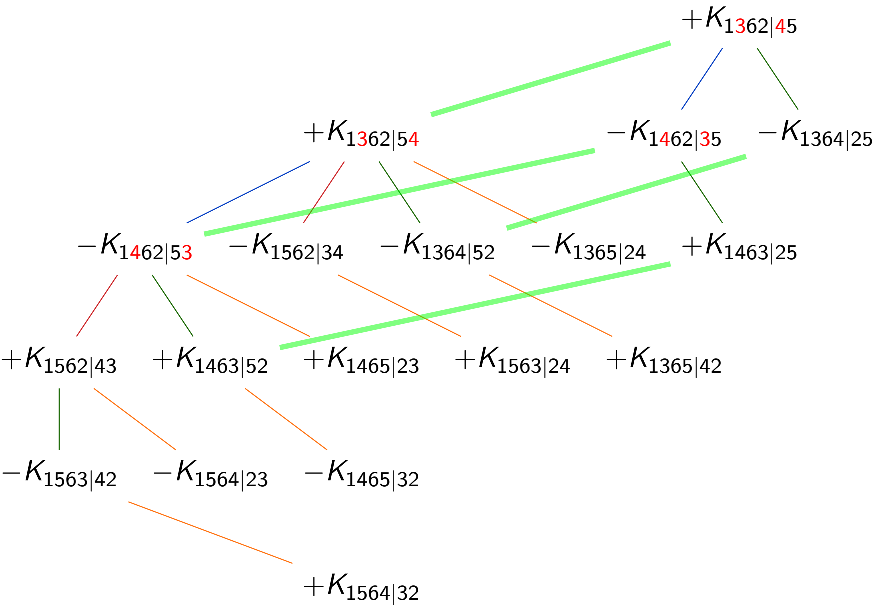
$+K_{1564|32}$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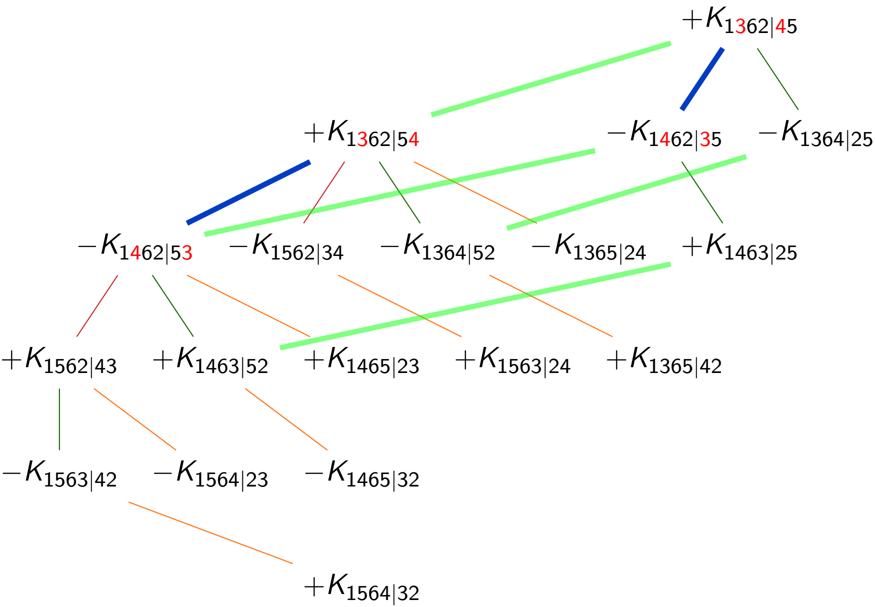
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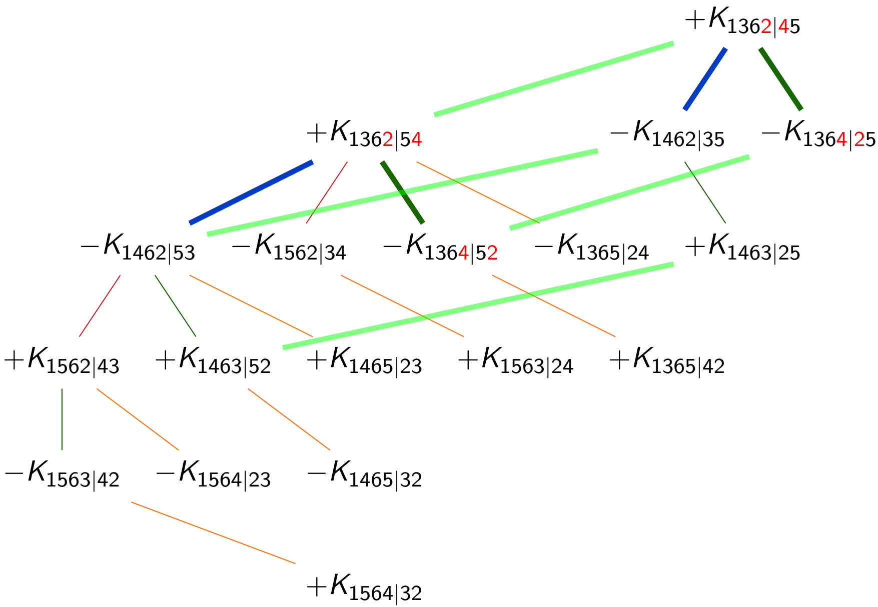
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$+K_{1362|45}$

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$-K_{1462|53}$ $\quad -K_{1562|34}$ $\quad -K_{1364|52}$ $\quad -K_{1365|24}$ $\quad +K_{1463|25}$

$+K_{1562|43}$ $\quad +K_{1463|52}$ $\quad +K_{1465|23}$ $\quad +K_{1563|24}$ $\quad +K_{1365|42}$

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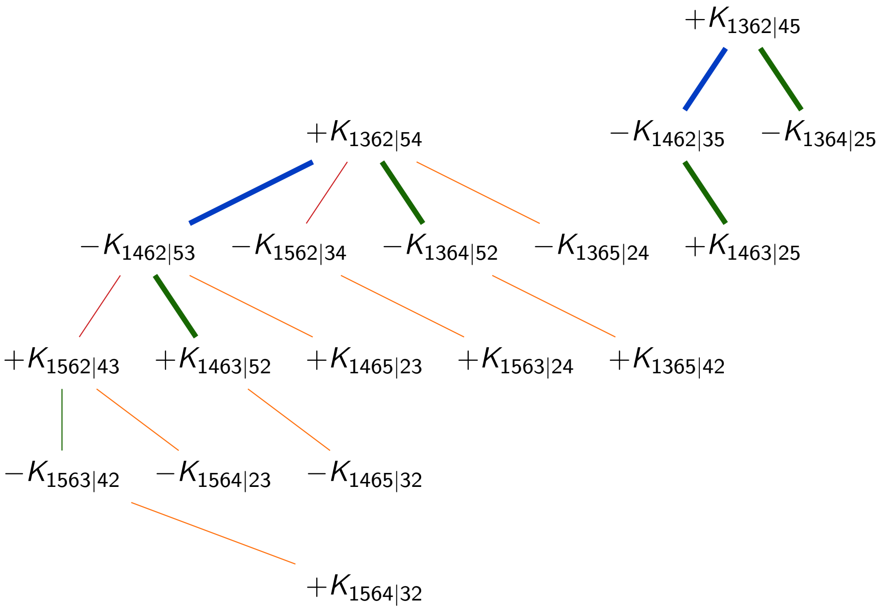
$+K_{1564|32}$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$+K_{1564|32}$

$+K_{1362|45}$

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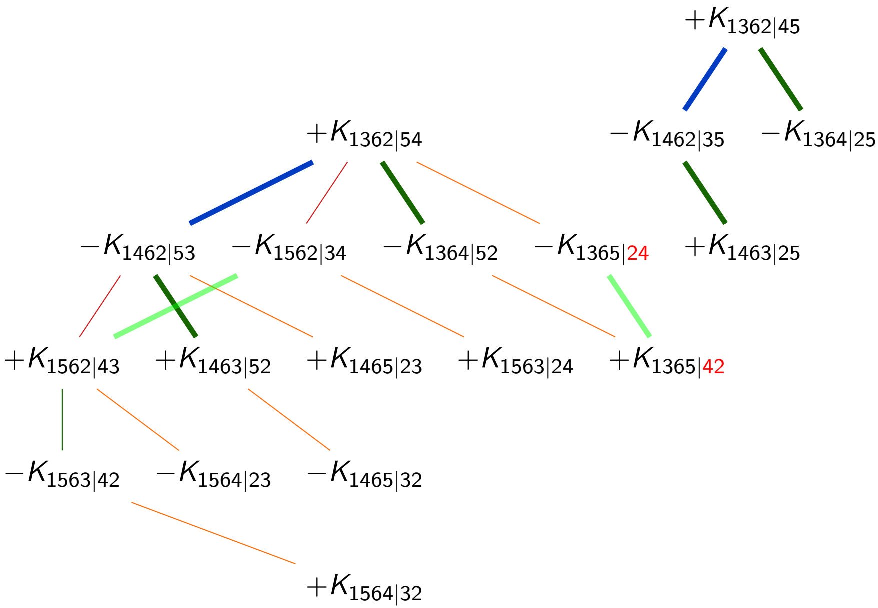
$+K_{1564|32}$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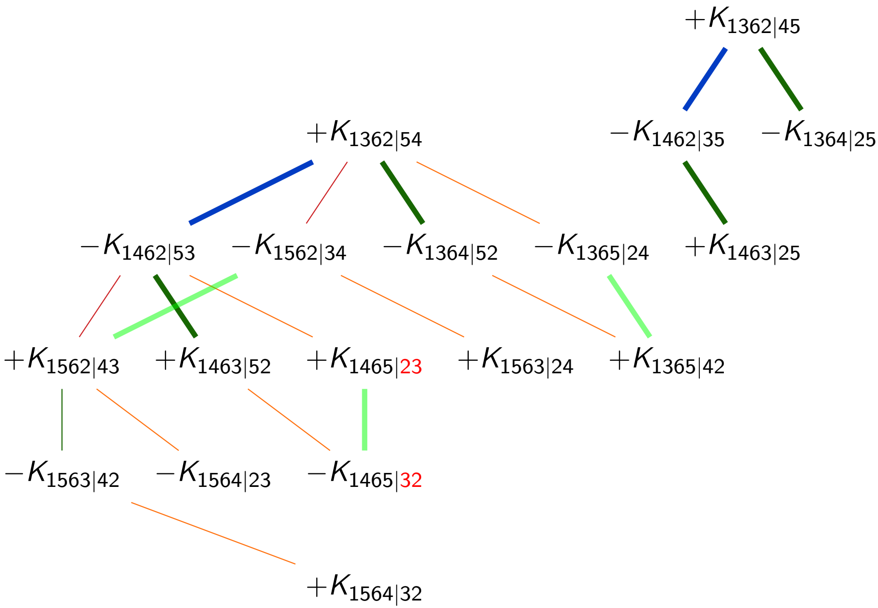
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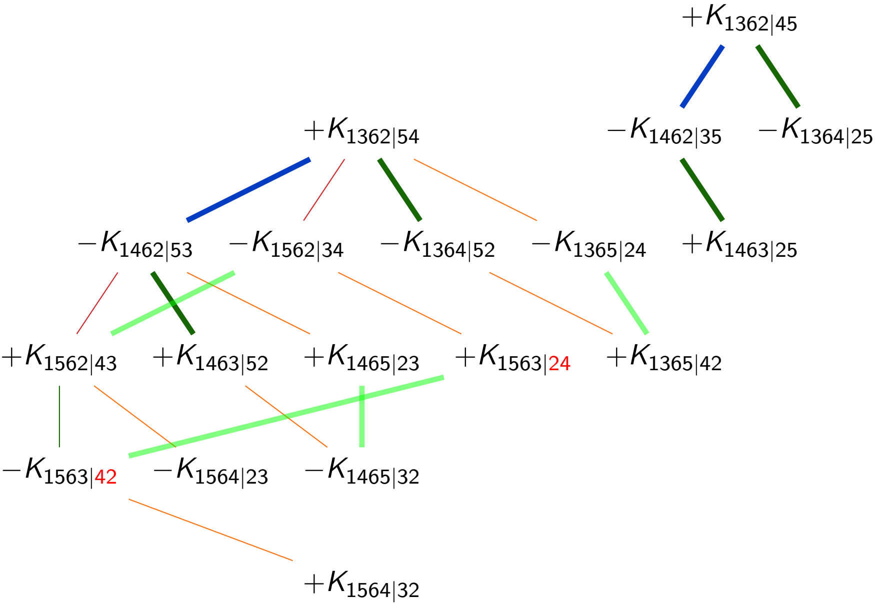
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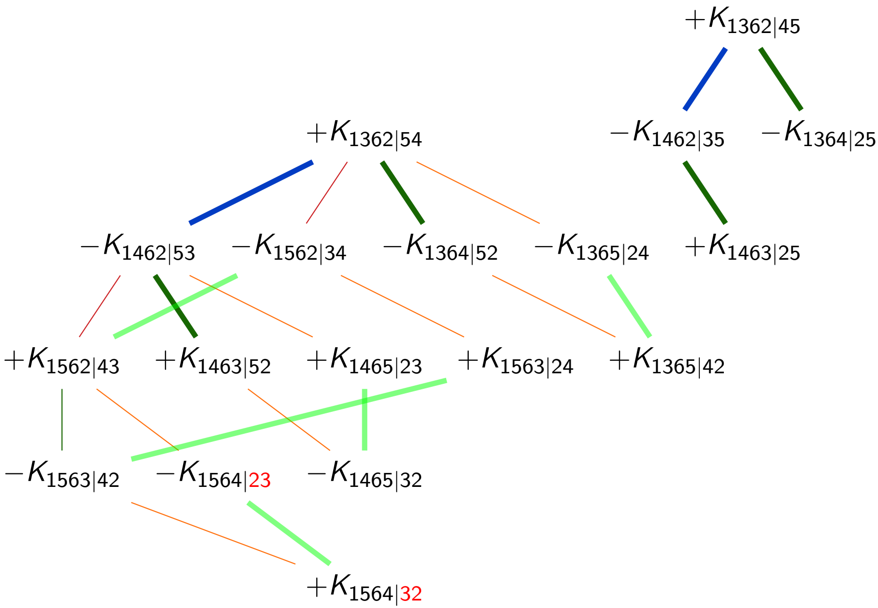
$-K_{1563|42}$
$-K_{1564|23}$
$-K_{1465|32}$
$+K_{1564|32}$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$-K_{1364|25}$
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$-K_{1562|34}$
$-K_{1364|52}$
$-K_{1365|24}$
$+K_{1562|43}$
$+K_{1463|52}$
$+K_{1465|23}$
$+K_{1563|24}$
$+K_{1365|42}$
$-K_{1563|42}$
$-K_{1564|23}$
$-K_{1465|32}$
$+K_{1564|32}$

$$+K_{1362|45}$$

$$+K_{1362|54}$$

$$-K_{1462|53} \quad -K_{1562|34} \quad -K_{1364|52} \quad -K_{1365|24}$$

$$+K_{1562|43} \quad +K_{1463|52} \quad +K_{1465|23} \quad +K_{1563|24} \quad +K_{1365|42}$$

$$-K_{1563|42} \quad -K_{1564|23} \quad -K_{1465|32}$$

$$+K_{1564|32}$$

$+K_{1362|45}$

$+K_{1362|54}$

$-K_{1462|53}$ $-K_{1562|34}$ $-K_{1364|52}$ $-K_{1365|24}$

$+K_{1562|43}$ $+K_{1463|52}$ $+K_{1465|23}$ $+K_{1563|24}$ $+K_{1365|42}$

$-K_{1563|42}$ $-K_{1564|23}$ $-K_{1465|32}$

$+K_{1564|32}$

$+K_{1362|45}$

$+K_{1362|54}$ $\quad$ $-K_{1462|35}$

$-K_{1462|53}$ $\quad$ $-K_{1562|34}$ $\quad$ $-K_{1364|52}$ $\quad$ $-K_{1365|24}$

$+K_{1562|43}$ $\quad$ $+K_{1463|52}$ $\quad$ $+K_{1465|23}$ $\quad$ $+K_{1563|24}$ $\quad$ $+K_{1365|42}$

$-K_{1563|42}$ $\quad$ $-K_{1564|23}$ $\quad$ $-K_{1465|32}$

$+K_{1564|32}$

$+K_{1362|45}$

$+K_{1362|54}$ $\quad -K_{1462|35}$

$-K_{1462|53}$ $\quad -K_{1562|34}$ $\quad -K_{1364|52}$ $\quad -K_{1365|24}$

$+K_{1562|43}$ $\quad +K_{1463|52}$ $\quad +K_{1465|23}$ $\quad +K_{1563|24}$ $\quad +K_{1365|42}$

$-K_{1563|42}$ $\quad -K_{1564|23}$ $\quad -K_{1465|32}$

$+K_{1564|32}$

$$+K_{1362|45}$$

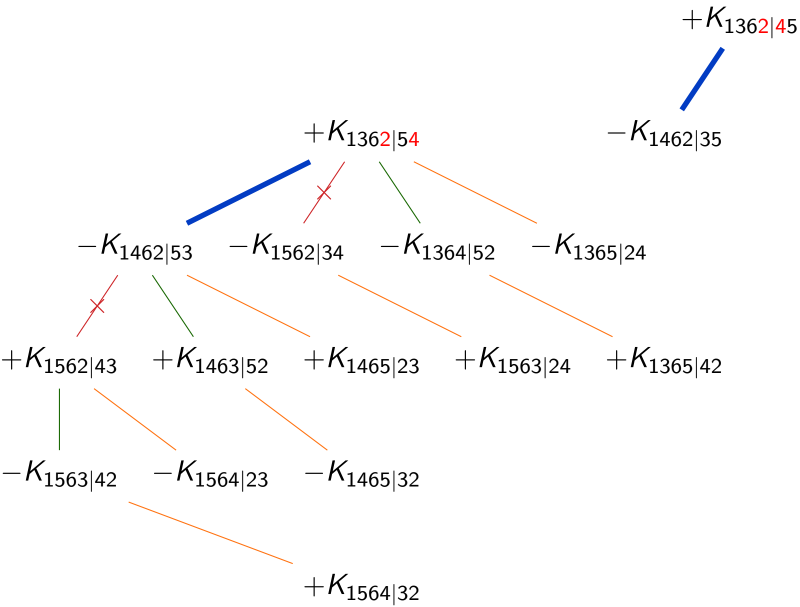
$$+K_{1362|54} \qquad -K_{1462|35}$$

$$-K_{1462|53} \quad -K_{1562|34} \quad -K_{1364|52} \quad -K_{1365|24}$$

$$+K_{1562|43} \quad +K_{1463|52} \quad +K_{1465|23} \quad +K_{1563|24} \quad +K_{1365|42}$$

$$-K_{1563|42} \quad -K_{1564|23} \quad -K_{1465|32}$$

$$+K_{1564|32}$$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$+K_{1362|54}$
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$-K_{1562|34}$
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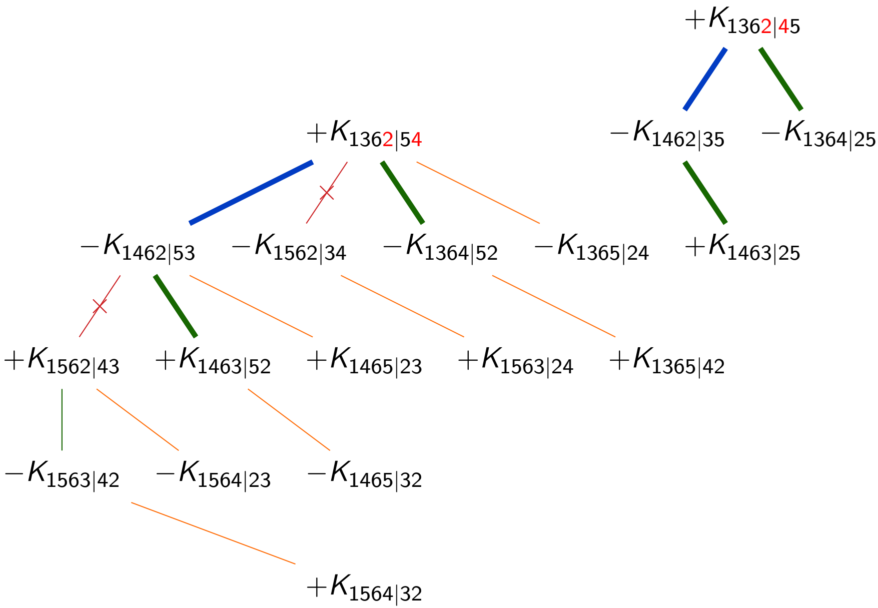
$+K_{1562|43}$
$+K_{1463|52}$
$+K_{1465|23}$
$+K_{1563|24}$
$+K_{1365|42}$
$-K_{1563|42}$
$-K_{1564|23}$
$-K_{1465|32}$
$+K_{1564|32}$


$+K_{1362|45}$
$+K_{1362|54}$
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$-K_{1562|34}$
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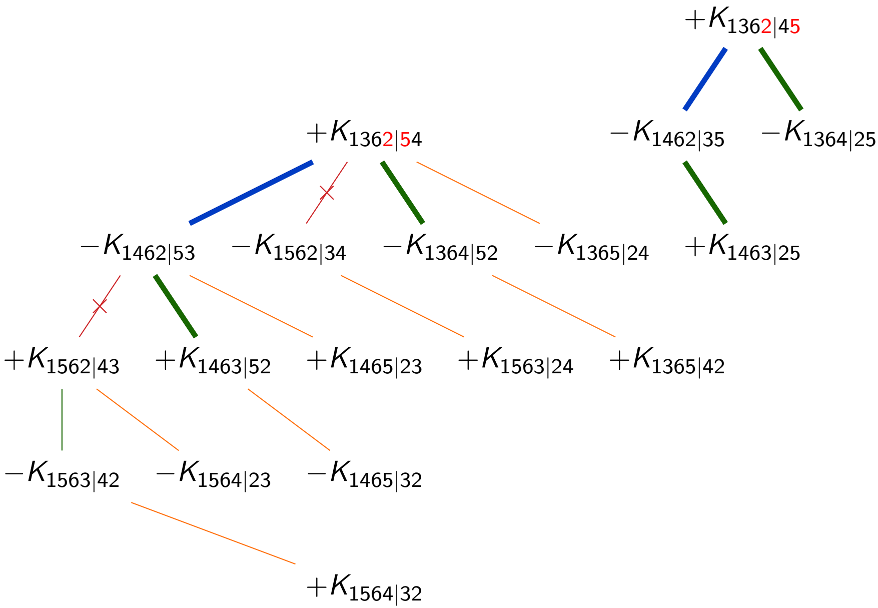
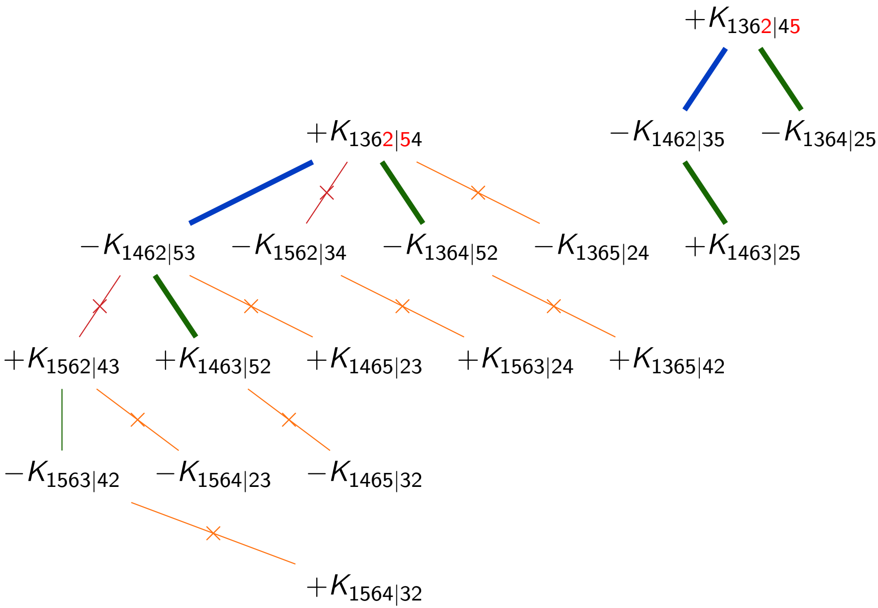
$-K_{1365|24}$
$+K_{1463|25}$
$+K_{1562|43}$
$+K_{1463|52}$
$+K_{1465|23}$
$+K_{1563|24}$
$+K_{1365|42}$
$-K_{1563|42}$
$-K_{1564|23}$
$-K_{1465|32}$
$+K_{1564|32}$


$+K_{1362|45}$
$+K_{1362|54}$
$-K_{1462|35}$
$-K_{1364|25}$
$-K_{1462|53}$
$-K_{1562|34}$
$-K_{1364|52}$
$-K_{1365|24}$
$+K_{1463|25}$
$+K_{1562|43}$
$+K_{1463|52}$
$+K_{1465|23}$
$+K_{1563|24}$
$+K_{1365|42}$
$-K_{1563|42}$
$-K_{1564|23}$
$-K_{1465|32}$
$+K_{1564|32}$


$+K_{1362|45}$
$-K_{1462|35}$
$-K_{1364|25}$
$+K_{1362|54}$
$-K_{1462|53}$
$-K_{1562|34}$
$-K_{1364|52}$
$-K_{1365|24}$
$+K_{1463|25}$
$+K_{1562|43}$
$+K_{1463|52}$
$+K_{1465|23}$
$+K_{1563|24}$
$+K_{1365|42}$
$-K_{1563|42}$
$-K_{1564|23}$
$-K_{1465|32}$
$+K_{1564|32}$

$$\zeta = 6321|54, \quad W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\zeta| = 46$$

$$\zeta = 6321|54, \quad W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\zeta| = 46$$
$$\downarrow s_3$$
$$\sigma = 6312|54, \quad W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), \quad |\mathfrak{E}_\sigma| = 14$$

$$
\begin{array}{lll}
\zeta = 6321|54, & W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\zeta| = 46 \\
\downarrow s_3 & & \\
\sigma = 6312|54, & W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_2 & & \\
\sigma = 6132|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14
\end{array}
$$

$$
\begin{array}{lll}
\zeta = 6321|54, & W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\zeta| = 46 \\
\downarrow s_3 & & \\
\sigma = 6312|54, & W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_2 & & \\
\sigma = 6132|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_1 & & \\
\sigma = 1632|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14
\end{array}
$$

$$\zeta = 6321|54, \quad W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\zeta| = 46$$

$$\downarrow s_3$$

$$\sigma = 6312|54, \quad W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), \quad |\mathfrak{E}_\sigma| = 14$$

$$\downarrow s_2$$

$$\sigma = 6132|54, \quad W_\sigma = ((3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\sigma| = 14$$

$$\downarrow s_1$$

$$\sigma = 1632|54, \quad W_\sigma = ((3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\sigma| = 14$$

$$\downarrow s_2$$

$$\sigma = 1362|54, \quad W_\sigma = ((2,6),(2,5),(4,6),(4,5)), \quad |\mathfrak{E}_\sigma| = 14$$

$$\zeta = 6321|54, \quad W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\zeta| = 46$$
$$\downarrow s_3$$
$$\sigma = 6312|54, \quad W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), \quad |\mathfrak{E}_\sigma| = 14$$
$$\downarrow s_2$$
$$\sigma = 6132|54, \quad W_\sigma = ((3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\sigma| = 14$$
$$\downarrow s_1$$
$$\sigma = 1632|54, \quad W_\sigma = ((3,6),(3,5),(4,6),(4,5)), \quad |\mathfrak{E}_\sigma| = 14$$
$$\downarrow s_2$$
$$\sigma = 1362|54, \quad W_\sigma = ((2,6),(2,5),(4,6),(4,5)), \quad |\mathfrak{E}_\sigma| = 14$$
$$\downarrow s_5$$
$$\sigma = 1362|45, \quad W_\sigma = ((2,5),\cancel{(2,6)},(4,5),\cancel{(4,6)}), \quad |\mathfrak{E}_\sigma| = 4$$

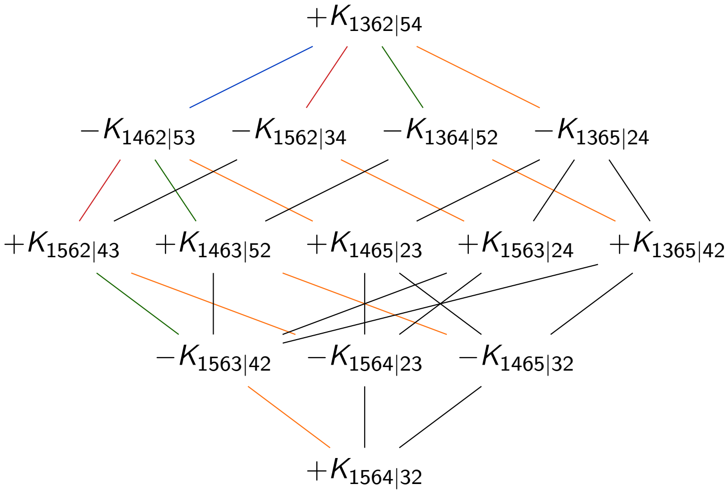
$$
\begin{array}{lll}
\zeta = 6321|54, & W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\zeta| = 46 \\
\downarrow s_3 & & \\
\sigma = 6312|54, & W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_2 & & \\
\sigma = 6132|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_1 & & \\
\sigma = 1632|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_2 & & \\
\sigma = 1362|54, & W_\sigma = ((2,6),(2,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_5 & & \\
\sigma = 1362|45, & W_\sigma = ((2,5),\cancel{(2,6)},(4,5),\cancel{(4,6)}), & |\mathfrak{E}_\sigma| = 4 \\
\downarrow s_3 & & \\
\sigma = 1326|45, & W_\sigma = ((2,5),(3,5)), & |\mathfrak{E}_\sigma| = 4
\end{array}
$$

$$
\begin{array}{lll}
\zeta = 6321|54, & W_\zeta = ((2,6),(2,5),(3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\zeta| = 46 \\
\downarrow s_3 & & \\
\sigma = 6312|54, & W_\sigma = ((2,6),(2,5),(4,6),(4,5),\cancel{(3,6)},\cancel{(3,5)}), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_2 & & \\
\sigma = 6132|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_1 & & \\
\sigma = 1632|54, & W_\sigma = ((3,6),(3,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_2 & & \\
\sigma = 1362|54, & W_\sigma = ((2,6),(2,5),(4,6),(4,5)), & |\mathfrak{E}_\sigma| = 14 \\
\downarrow s_5 & & \\
\sigma = 1362|45, & W_\sigma = ((2,5),\cancel{(2,6)},(4,5),\cancel{(4,6)}), & |\mathfrak{E}_\sigma| = 4 \\
\downarrow s_3 & & \\
\sigma = 1326|45, & W_\sigma = ((2,5),(3,5)), & |\mathfrak{E}_\sigma| = 4 \\
\downarrow s_2 & & \\
\sigma = 1236|45, & W_\sigma = ((3,5),\cancel{(2,5)}), & |\mathfrak{E}_\sigma| = 2
\end{array}
$$

Structure :

$$
\begin{tikzcd}[column sep=tiny]
 & & & & +K_{1362|54} \arrow[dlll, dash] \arrow[dl, dash] \arrow[dr, dash] \arrow[drrr, dash] & & & & \\
 & -K_{1462|53} \arrow[dl, dash] \arrow[dr, dash] \arrow[drrr, dash] & & -K_{1562|34} \arrow[drrr, dash] & & -K_{1364|52} \arrow[drrr, dash] & & -K_{1365|24} & \\
+K_{1562|43} \arrow[drr, dash] \arrow[drrrr, dash] & & +K_{1463|52} \arrow[drrrr, dash] & & +K_{1465|23} & & +K_{1563|24} & & +K_{1365|42} \\
 & & -K_{1563|42} \arrow[drr, dash] & & -K_{1564|23} & & -K_{1465|32} & & \\
 & & & & +K_{1564|32} & & & &
\end{tikzcd}
$$

Structure : Intervalle



Merci de votre attention.