What is Sage ?
A patch for multi-bases polynomials
Live demo
Where is it ? How do I install it ?

Multivariate Polynomials in Sage

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What is Sage ?

- A mathematics software

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What is Sage ?

- ▶ A mathematics software
- ▶ Free and open-source

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What is Sage ?

- ▶ A mathematics software
- ▶ Free and open-source
- ▶ Combining the power of many open-sources packages : Atlas, GAP, GMP, Linbox, Maxima, MPFR, PARI/GP, NetworkX, NTL, Numpy/Scipy, Singular, Symmetrica, ...

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- ▶ Written in Python

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Some numbers

► Created in 2005

► 184 developers

► 10000 users

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Implementing multivariate polynomials

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Implementing multivariate polynomials as a multi-bases algebra

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We define a monomial by :

$$x^v := x_1^{v_1} x_2^{v_2} \ldots x_n^{v_n}$$

where $v = (v_1, \ldots, v_n) \in \mathbb{Z}^n$

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From an operation on the vector :

$$vs_i = (\ldots, v_{i+1}, v_i, \ldots)$$

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From an operation on the vector :

$$vs_i = (\ldots, v_{i+1}, v_i, \ldots)$$

we get an operation on the monomial :

$$x^v s_i = x^{vs_i}$$

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From the basic operators s_i, we get the *divided differences* operators : ∂_i, π_i, $\hat{\pi}_i$, T_i

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And from the operators, we get linear basis :

$$Y_{210} = x^{(2,1,0)}$$

$\partial_1 \swarrow \qquad \searrow \partial_2$

$$Y_{110} = x^{(1,1,0)} \qquad\qquad Y_{200} = x^{(2,0,0)}$$

$\partial_2 \downarrow \qquad\qquad\qquad \downarrow \partial_1$

$$Y_{100} = x^{(1,0,0)} \qquad\qquad Y_{010} = x_1 + x_2$$

$\partial_1 \searrow \qquad\qquad \swarrow \partial_2$

$$Y_{000} = 1$$

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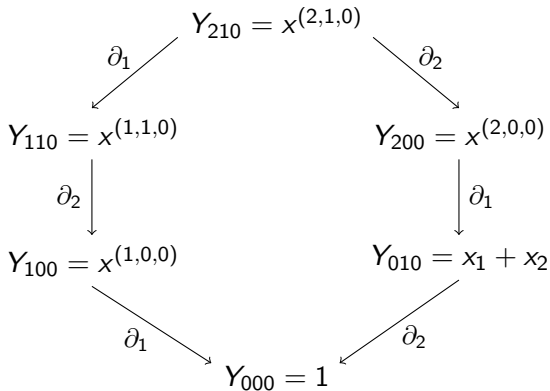
Implemented basis :

- ▶ Schubert simple and double basis
- ▶ Key polynomials in types A,B,C,D
- ▶ Grothendieck polynomials
- ▶ Macdonald non symmetric polynomials

Main features

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Main features

- Define polynomials in a set of indexed variables

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Main features

- ▶ Define polynomials in a set of indexed variables
- ▶ Apply divided differences operators

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Main features

- ▶ Define polynomials in a set of indexed variables
- ▶ Apply divided differences operators
- ▶ Convert into other basis, compute in the basis you want

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Main features

- ▶ Define polynomials in a set of indexed variables
- ▶ Apply divided differences operators
- ▶ Convert into other basis, compute in the basis you want
- ▶ Work with a double set of variables and double linear basis

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Main features

- ▶ Define polynomials in a set of indexed variables
- ▶ Apply divided differences operators
- ▶ Convert into other basis, compute in the basis you want
- ▶ Work with a double set of variables and double linear basis
- ▶ Define your own basis and let the conversion system work

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Let's see the demo !

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Where to find this patch ?

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Where to find this patch ?

This patch is still a **work in progress**. It is not available yet on the main Sage distribution. You may find it on **Sage-Combinat** distribution.

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What is Sage-Combinat ?

Sage-combinat is a collection of experimental patches about combinatorics. The patches are reviewed tested by the Sage-Combinat community and eventually added to the main Sage distribution.

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How to install it ?

- ▶ Install Sage [1]
- ▶ Install the Sage-Combinat distribution [3] by typing
  sage -combinat install

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Thank you for your attention.

📄 The Sage website
http ://www.sagemath.org/

📄 [Janvier 2011, Orsay] Sage Days 28 : Dynamics, geometry, combinatorics...
Laboratoire de Mathmatiques d'Orsay, Université Paris Sud
http ://wiki.sagemath.org/days28

📄 The Sage-Combinat website
http ://wiki.sagemath.org/combinat

📄 This presentation and the demo file
http ://www-igm.univ-mlv.fr/~pons